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SUBLUMINAL AND SUPERLUMINAL UNIVERSES

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Abstract: We compare a closed positive curved and an open negative curved expanding universe. The physical velocities and the Hubble equation are calculated. We demonstrate that in an open universe, the physically defined velocities certainly exceed the velocity of light. In addition, we show that the recession velocities cannot be an effect of expansion of space but is an ordinary physical velocity. The open universes violate the principles of Special Relativity and are ruled out from being realized by Nature.

1. INTRODUCTION

In a former paper [1] we derived a cosmological model that expands linearly, is closed, and in it the recession velocity does not exceed the velocity of light. We derived our model from the dS model by excluding the restriction $\mathcal{R}_0 = \text{const.}$, $\mathcal{R}_0$ being the radius of the pseudo-hypersphere, geometrically representing the structure of the dS universe.

In this paper, we supplement the present model with a similar model concerning the geometrical structure; however, this model has a negative curvature. The first model, with a positive curvature, is called the Subluminal Model (I). The second model that has a negative curvature is called the Superluminal Model (II). These models have been named according to their behavior concerning the recession velocity of the galaxies. The Subluminal Model was derived from the static dS model, and the Superluminal Model we derive from the AdS (anti-de Sitter) model. For both models, we exclude the condition $\mathcal{R} = \text{const.}$. The formal working through of these models is very similar. The mathematical tools can be found in paper [1]. We have presented numerous calculations in the appendix in order to obtain a better overview of the structures of the models.

We only use physical quantities in our paper that can be measured with standard rods and clocks. These are represented by tetrads. The inhomogeneous law of transformation of the Ricci-rotation coefficients provides a transition of reference systems

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to relatively moving ones. Moreover, we apply the original Minkowski notation $x^4 = i(c)t$ as it is the only one that allows a correct processing of the tetrads with the Ricci-rotation coefficients.

2. THE METRIC OF TWO STATIC MODELS

First, we recall the basic elements of the dS and AdS models, which will serve as auxiliary models for expanding universes. Both have embeddings (A1), i.e., they are based on pseudo-hyperspheres embedded into 5-dimensional flat spaces. The spatial part of AdS can be deduced from the dS by the substitution $\mathcal{R}_0 \to i\mathcal{R}_0, \eta \to i\eta$. The $\mathcal{R}_0$ are the constant radii and the η are the polar angles of the pseudo-hyperspheres.

The line elements for both cases are as follows:

(I B) $$ds^2 = \mathcal{R}_0^2 d\eta^2 + \mathcal{R}_0^2 \sin^2\eta d\vartheta^2 + \mathcal{R}_0^2 \sin^2\eta \sin^2\vartheta d\varphi^2 + \mathcal{R}_0^2 \cos^2\eta di\psi^2 \quad (2.1)$$

(I B') $$ds^2 = \frac{1}{\cos^2\eta} dr^2 + r^2 d\vartheta^2 + r^2 \sin^2\vartheta d\varphi^2 - \cos^2\eta dt^2 \quad (2.2)$$

(I B'') $$ds^2 = \frac{1}{1-\frac{r^2}{\mathcal{R}_0^2}} dr^2 + r^2 d\vartheta^2 + r^2 \sin^2\vartheta d\varphi^2 - \left(1-\frac{r^2}{\mathcal{R}_0^2}\right) dt^2 \quad (2.3)$$

(II B) $$ds^2 = \mathcal{R}_0^2 d\eta^2 + \mathcal{R}_0^2 \mathrm{sh}^2\eta d\vartheta^2 + \mathcal{R}_0^2 \mathrm{sh}^2\eta \sin^2\vartheta d\varphi^2 + \mathcal{R}_0^2 \mathrm{ch}^2\eta di\psi^2 \quad (2.4)$$

(II B') $$ds^2 = \frac{1}{\mathrm{ch}^2\eta} dr^2 + r^2 d\vartheta^2 + r^2 \sin^2\vartheta d\varphi^2 - \mathrm{ch}^2\eta dt^2 \quad (2.5)$$

(II B'') $$ds^2 = \frac{1}{1+\frac{r^2}{\mathcal{R}_0^2}} dr^2 + r^2 d\vartheta^2 + r^2 \sin^2\vartheta d\varphi^2 - \left(1+\frac{r^2}{\mathcal{R}_0^2}\right) dt^2. \quad (2.6)$$

Here, we have used the relations $r = \mathcal{R}_0 \sin\eta$, $r = \mathcal{R}_0 \mathrm{sh}\eta$, and $idt = \mathcal{R}_0 di\psi$. $\mathcal{R}, \eta, \vartheta, \varphi, \psi$ are the quasi-polar coordinates of the pseudo-hyperspheres.

(I B'') and (II B'') are written in the canonical form. From these, it is possible to read the type of curvature, i.e., $k = 1$ for the dS and $k = -1$ for the AdS model. k is called curvature parameter. Thus, the dS is closed and positively curved, and AdS is open and negatively curved.

Moreover, the points (galaxies) receding/oncoming in radial directions driven by the gravitational forces (A2) deduced from the lapse functions of the metrics can be read from B''. The velocities are $v = r/\mathcal{R}_0 = \sin\eta$, $v = r/\mathcal{R}_0 = \mathrm{sh}\eta$. For the dS model, the highest velocity is the velocity of light, which can only be reached asymptotically. It is the velocity at the equator of the pseudo-hypersphere $r = \mathcal{R}_0, \eta = \pi/2$. For the open AdS, the quantity r is unbounded, the velocities of the galaxies can become arbitrarily high. Consequently, the principles of Special Relativity are violated. Therefore, an expanding model based on AdS is ruled out for physical interpretation. Nevertheless, its mathematical structure is of significant interest.

Using the expressions for the velocities, the pseudo-rotations (A3) for both models can be easily constructed. With the inhomogeneous transformation law of the Ricci-rotation coefficients (A4), one can transform the radial field strengths (A2) from the static systems into the systems comoving with the drifting particles by evaluating the 2nd Term of (A4). Therefore, [2][1] is obtained:

$$\begin{aligned}
&(\mathrm{I}) \quad U_m = \left\{ -\frac{1}{\mathcal{R}_0} \tan\eta, 0,0,0 \right\} \rightarrow {}'U_{m'} = \left\{ 0,0,0,-\frac{i}{\mathcal{R}_0} \right\} \\
&(\mathrm{II}) \quad U_m = \left\{ \frac{1}{\mathcal{R}_0} \mathrm{th}\eta, 0,0,0 \right\} \rightarrow {}'U_{m'} = \left\{ 0,0,0,\frac{1}{\mathcal{R}_0} \right\}
\end{aligned} \qquad (2.7)$$

These are the fundamental structures that can be the base of expanding/contracting models. The static quantities have a single component: the radial one – the force of gravity. This force vanishes in comoving systems. Instead, a time-like component appears in the corresponding quantities of comoving systems. These are the tidal forces describing the expansion/contraction of the drifting particle clouds. Moreover, the lateral field quantities [2] receive the same fourth components. The particle clouds expand/contract into all three spatial directions in an equal manner, which could easily be described by evaluating the expansion scalar. The three spatial components of the lateral field quantities describe an apparently flat space.

The metrics (I) and (II) written in non-comoving coordinates can be transformed into metrics in comoving coordinates with the help of $\overset{m'}{e}_{i'} = L^{m'}_{m} \overset{m}{e}_{i} \Lambda^{i}_{i'}$; L is the pseudo-rotation (A3) operating on the tetrads. The coordinate transformations Λ can be found in appendix (A5). It is noteworthy that the transformation (A5,II) is a Lemaître transformation; however, it is not a Florides transformation [6]. The latter would lead us to a FRW type of comoving metric for the AdS. However, we have no confidence in this type of metric. From the aforementioned relations, the metrics in non-comoving coordinates can be obtained:

$$(\mathrm{AI}) \quad ds^2 = \mathcal{K}^2 \left(dr'^2 + r'^2 d\vartheta^2 + r'^2 \sin^2\vartheta d\varphi^2 \right) - dt'^2 , \qquad (2.8)$$

$$(\mathrm{AII}) \quad ds^2 = \mathcal{K}^2 \left(dr'^2 + r'^2 d\vartheta^2 + r'^2 \sin^2\vartheta d\varphi^2 \right) - dt'^2 . \qquad (2.9)$$

Here, $\mathcal{K} = \mathcal{K}(t')$ is the time-dependent scale factor, describing the expansion of the particle cloud in comoving coordinates and t' the universal cosmological time, identical with the proper time of the drifting particles. Although the line elements are formally identical, they differ in terms of the definitions of r' and $\mathcal{K}$. Evidently, the metrics (A) are of type $k = 0$, typical for a flat space. This does not imply that curved spaces with $k = 1$ and $k = -1$ can be transformed with the help of a pseudo-rotation into a *globally flat* space but only into a *locally flat* space, according to Einstein's elevator principle [3][4][5]. The lapse functions in (AI), (AII) are $g_{4'4'} = 1$ and indicate that the cloud of particles moves in free fall. No gravitational forces emerge from these. Once again, the quantities $'U$ in (2.7) can be derived from the metrics (A). The $'U$ are to be determined by the Friedman equation, which is the fourth component of Einstein's field equations. Owing to Gauss, it is known that a line element can only describe the curvature of space but not the change of curvature. Therefore, a second set of equations is necessary to determine the quantities $'U$. These are the Bianchi identities which lead to the conservation laws.

First, there is $'U_{4'} = {}'A_{1'4}{}^{1'} = -\overset{1'}{e}_{1'} e^{1'}_{1'|4'} = \frac{1}{\mathcal{K}} \mathcal{K}_{|4'}$. Comparing this result with (2.7) the following is obtained for the dS

[1] A prime denotes the comoving system.

$$\text{(I)} \qquad \frac{1}{\mathcal{K}}\mathcal{K}_{|4'} = \frac{1}{\mathcal{K}}\frac{\partial}{i\partial t'}\mathcal{K} = -\frac{i}{\mathcal{R}_0}, \quad \mathcal{K} = e^{t'/\mathcal{R}_0} \tag{2.10}$$

in agreement with the results of Lemaître [7][8]. And for the AdS

$$\text{(II)} \qquad \frac{1}{\mathcal{K}}\mathcal{K}_{|4'} = \frac{1}{\mathcal{K}}\frac{\partial}{i\partial t'}\mathcal{K} = \frac{1}{\mathcal{R}_0}, \quad \mathcal{K} = e^{it'/\mathcal{R}_0}\,. \tag{2.11}$$

In addition, the relation between the non-comoving and comoving radial coordinates is $r = \mathcal{K}r'$. Therefore, the line elements (2.8) and (2.9) are elucidated. The field equations of the dS model are well known in the existing literature and are presented by us in the less known tetrad form in [1][2]. Concerning the AdS model, there does not seem to exist any literature for the Lemaître representation. Therefore, we have added some remarks concerning the field quantities and field equations in the appendix.

The EOS $p + \mu_0 = 0$ emerges for both models, p being the pressure of the cosmic fluid and μ_0 being its mass density. This has the consequence that no matter current occurs in the non-comoving reference system. This was discussed by Mitra [9] and Gron [10] in detail. We do not believe that this is an issue because both models are not very close to Nature. The dS has a negative pressure and positive mass density, and the AdS has a positive pressure and negative mass density.

3. TIME DEPENDENT MODELS

From the above-mentioned static models, we derive time-dependent models by excluding the restriction that the radii of the pseudo-hyperspheres are constant. We introduce the quantity

$$\mathcal{R} = \mathcal{R}(t'), \quad \mathcal{R} = \mathcal{K}\mathcal{R}_0\,, \tag{3.1}$$

wherein the $\mathcal{R}_0$ are constants. They are the radii of the pseudo-hyperspheres measured with expanding rods by observers comoving with the expansion of the pseudo-hyperspheres. $\mathcal{R}$ are the radii measured by non-comoving observers. Therefore, the new models comprise a series of self-similar dS and AdS universes describing the expansion of space. The occurrence of time-dependent quantities with

$$\frac{1}{\mathcal{R}}\mathcal{R}_{|4'} = \frac{1}{\mathcal{K}}\mathcal{K}_{|4'} \tag{3.2}$$

changes the structures of the subequations of the Ricci and the Friedman equations (A9, A10). (3.2) has the following as solutions:

$$\text{(I)} \quad \mathcal{R}^{\cdot} = 1, \quad \mathcal{R}^{\cdot\cdot} = 0, \quad \text{(II)} \quad (i\mathcal{R})^{\cdot} = 1, \quad \mathcal{R}^{\cdot\cdot} = 0\,. \tag{3.3}$$

This indicates that both models are expanding linearly. The linear expansions are aided by recent evaluations of astrophysical data by Melia, Quin, and Zhang [11]. He and his coworkers reexamined the data of the SDSS-IV Quasar Catalog with the Alcock-Paczyński effect. They concluded that Melia's R_h=ct model is favored for explaining these data. We also want to draw the reader's attention to a paper by Krasiński [12] ("*Cosmological models and misunderstanding about them.*"). He wrote, "*The accelerating expansion of the Universe is not an observed phenomenon, but an element of interpretation of observations, forced upon us by the R-W models*".

From Einstein's field equations and the Bianchi identities, we can obtain the time-dependent quantities

$$\text{(I)}\quad \kappa p = -\frac{1}{\mathcal{R}^2},\quad \kappa\mu_0 = \frac{3}{\mathcal{R}^2},\quad \text{(II)}\quad \kappa p = \frac{1}{\mathcal{R}^2},\quad \kappa\mu_0 = -\frac{3}{\mathcal{R}^2}. \tag{3.4}$$

We note that both models have the same EOS $3p+\mu_0=0$. Consequently, currents appear in the non-comoving systems. This is expected for physically interpretable models.

The problem of recession velocities is of significant interest. Usually, they are evaluated with the Hubble law:

$$r^{\bullet} = Hr,\quad H = \frac{\mathcal{R}^{\bullet}}{\mathcal{R}} = \frac{\mathcal{K}^{\bullet}}{\mathcal{K}},\quad r^{\bullet} = \frac{\partial r}{\partial t'}. \tag{3.5}$$

Evidently, the question of how the recession velocity can exceed the velocity of light is connected to the Hubble law. For our Subluminal Model [1] based on dS, the radial distance r is bounded, i.e., r has its highest value at the equator of the pseudo-hypersphere $r=\mathcal{R}$. Moreover, considering (3.3), $v=r^{\bullet}=\sin\eta$ is obtained. The cosmic horizon and the recession velocity are defined geometrically. This is not the case for a model based on the AdS. Here, the radial coordinate r is unbounded and the recession velocity consequently admits superluminal values. The cosmic horizon must be introduced artificially.

In the existing literature, one finds several arguments to discuss superluminal effects.

First, some authors claim that the recession velocity could be a coordinate velocity but not a physical velocity and therefore admits superluminal values. Indeed, considering the Hubble law (3.5), it is recognizable that this is not an invariant equation. It consists of the non-comoving quantity r and the comoving cosmic time t'. In [1], we have shown that the law can be reformulated as an invariant structure. Owing to the fact that the spatial parts of the dS/AdS model and the Subluminal/Superluminal Model coincide at any time, we get $dx^1=\alpha dr,\quad \alpha=1/\cos\eta;\quad \alpha=1/\mathrm{ch}\,\eta$. Further, the relation between the proper times of the non-comoving and comoving systems are known to be $dT/dT'=dT/dt'=\alpha$. α is the Lorentz factor for (I) as well as for (II) in the causal region. Finally, the following relation is obtained:

$$v = r^{\bullet} \to v = \frac{dx^1}{dT}. \tag{3.6}$$

Thus, the recession velocities are physically defined quantities.

Another argument that the recession velocity does not violate Special Relativity is made by assuming that the velocity is not the velocity of the particles but an effect of expansion. This is evidently not the case for our two examples. The recession velocities are the velocities of the drifting particles of the static seed models. In contrast, the expansion is introduced in such a manner that that the expansion velocities coincide with the drifting velocities of the static models and are thus physical velocities. Indeed, Special Relativity breaks down for open models with a negative curvature.

Furthermore, we will examine Melia's R_h=ct model. Melia's model is based on a metric of type (2.8) with the curvature parameter $k=0$. Thus, Melia is of the opinion that his model is *globally flat* and infinite. He introduces the cosmic horizon by borrowing the ideas of an event horizon from the Schwarzschild theory. Ignoring Einstein's elevator principle [4][5][13], he fails to realize that his model is only *locally flat* and essentially identical to our Subluminal Model [1]. Melia's grandiose research on astrophysical data supports his model and equally supports our Subluminal Model.

Finally, we will initiate an interesting question concerning infinite models, a question for which we did not find an answer in the existing literature. An infinitely large amount of matter in the universe produces attractive gravitational fields all over the space. Although, the forces decrease with the inverse second power law; the question is how they sum up in our orbital system. Could we expect a gravitational Olbers' paradox? Or do expanding effects resolve this problem?

4. CONCLUSIONS

We transformed the static metric of the AdS model into a Lemaître form. This metric evidently differs from the FRW structure which could be obtained with a Florides transformation. The model has some peculiar features which cannot be attributed to physical properties. Nevertheless we extended this model to an expanding model. We aimed to compare the structure of this model with that of our Subluminal Model derived from the static dS model. We encountered recession velocities and demonstrated that these are physical quantities adopted from the velocities of the drifting points of the static version. Moreover, the recession velocities cannot be explained as an expansion effect.

Since the superluminal velocities in the AdS version cannot be discussed away, this model is physically not acceptable. We argue that this also applies to other open infinite models.

5. MATHEMATICAL APPENDIX

1. The following are the embeddings for the dS and AdS models:

$$\text{(I)}\quad \begin{aligned} x^3 &= \mathcal{R}_0 \sin\eta \sin\vartheta \sin\varphi \\ x^2 &= \mathcal{R}_0 \sin\eta \sin\vartheta \cos\varphi \\ x^1 &= \mathcal{R}_0 \sin\eta \cos\vartheta \\ x^4 &= \mathcal{R}_0 \cos\eta \sin i\psi \\ x^0 &= \mathcal{R}_0 \cos\eta \cos i\psi \end{aligned} \qquad \text{(II)}\quad \begin{aligned} x^3 &= \mathcal{R}_0 \mathrm{sh}\eta \sin\vartheta \sin\varphi \\ x^2 &= \mathcal{R}_0 \mathrm{sh}\eta \sin\vartheta \cos\varphi \\ x^1 &= \mathcal{R}_0 \mathrm{sh}\eta \cos\vartheta \\ x^4 &= i\mathcal{R}_0 \mathrm{ch}\eta \sin\psi \\ x^0 &= i\mathcal{R}_0 \mathrm{ch}\eta \cos\psi \end{aligned} \qquad \text{(A1)}$$

2. Gravitational forces:

Using $dx^1 = \mathcal{R} d\eta$ one obtains $\eta_{|1} = 1/\mathcal{R}_0$. If one extracts the tetrads $\overset{4}{e}_4 = \cos\eta$, $\underset{4}{e}^4 = 1/\cos\eta$, $\overset{4}{e}_4 = \mathrm{ch}\eta$, $\underset{4}{e}^4 = 1/\mathrm{ch}\eta$ from the lapse function of (IB'), (IIB'), the gravitational forces[2]

$$\text{(I)}\quad U_m = \left\{ -\frac{1}{\mathcal{R}_0} \tan\eta, 0, 0, 0 \right\}, \quad \text{(II)}\quad U_m = \left\{ \frac{1}{\mathcal{R}_0} \mathrm{th}\eta, 0, 0, 0 \right\} \qquad \text{(A2)}$$

are derived via the Ricci-rotation coefficients [2].

3. The pseudo-rotations are as follows:

[2] The quantities U are the geometrical forces, the quantities -U the physical components.

$$\text{(I)}\quad \alpha = \frac{1}{\cos\eta} = 1\Big/\sqrt{1-r^2/\mathcal{R}_0^2}\,, \qquad\qquad \text{(II)}\quad \alpha = \frac{1}{\mathrm{ch}\eta} = 1\Big/\sqrt{1+r^2/\mathcal{R}_0^2}\,.$$

$$\text{(I)}\;\; L_{m'}^{m} = \begin{pmatrix} \frac{1}{\cos\eta} & & & -i\tan\eta \\ & 1 & & \\ & & 1 & \\ i\tan\eta & & & \frac{1}{\cos\eta} \end{pmatrix} \quad \text{(II)}\;\; L_{m'}^{m} = \begin{pmatrix} \frac{1}{\mathrm{ch}\eta} & & & \mathrm{th}\eta \\ & 1 & & \\ & & 1 & \\ -\mathrm{th}\eta & & & \frac{1}{\mathrm{ch}\eta} \end{pmatrix}. \qquad \text{(A3)}$$

Evidently, (I) is a Lorentz transformation; (II) does not fulfill the requirements for a physical transformation.

4. The inhomogeneous transformation law of the Ricci-rotation coefficients

$$'A_{m'n'}{}^{s'} = L_{m'n's}^{m\,n\,s'} A_{mn}{}^{s} + 'L_{m'n'}{}^{s'}, \quad 'L_{m'n'}{}^{s'} = L_{s}^{s'} L_{n'\,|m'}^{s} \qquad \text{(A4)}$$

can be used for transforming non-comoving Ricci-rotation coefficients A into comoving 'A ones.

5. The coordinate transformations of Lemaître type are

$$\Lambda_{i'}^{i} = \begin{pmatrix} \mathcal{K} & & & -i\sin\eta \\ & 1 & & \\ & & 1 & \\ i\mathcal{K}\frac{\sin\eta}{\cos^2\eta} & & & \frac{1}{\cos^2\eta} \end{pmatrix}, \qquad \Lambda_{i}^{i'} = \begin{pmatrix} \frac{1}{\mathcal{K}\cos^2\eta} & & & \frac{i\sin\eta}{\mathcal{K}} \\ & 1 & & \\ & & 1 & \\ -i\frac{\sin\eta}{\cos^2\eta} & & & 1 \end{pmatrix} \qquad \text{(A5,I)}$$

$$\Lambda_{i'}^{i} = \begin{pmatrix} \mathcal{K} & & & \mathrm{sh}\eta \\ & 1 & & \\ & & 1 & \\ -\mathcal{K}\frac{\mathrm{sh}\eta}{\mathrm{ch}^2\eta} & & & \frac{1}{\mathrm{ch}^2\eta} \end{pmatrix}, \qquad \Lambda_{i}^{i'} = \begin{pmatrix} \frac{1}{\mathcal{K}\,\mathrm{ch}^2\eta} & & & -\frac{\mathrm{sh}\eta}{\mathcal{K}} \\ & 1 & & \\ & & 1 & \\ \frac{\mathrm{sh}\eta}{\mathrm{ch}^2\eta} & & & 1 \end{pmatrix} \qquad \text{(A5,II)}$$

including the scale factor $\mathcal{K}$.

6. The field quantities and field equations of the AdS model in comoving coordinates are as follows:

$$B_{m'} = \left\{\frac{1}{r},0,0,\frac{1}{\mathcal{R}_0}\right\}, \quad C_{m'} = \left\{\frac{1}{r},\frac{1}{r}\cot\vartheta,0,\frac{1}{\mathcal{R}_0}\right\}, \quad 'U_{m'} = \left\{0,0,0,\frac{1}{\mathcal{R}_0}\right\}, \qquad \text{(A6)}$$

$$
\begin{aligned}
&{}'U_{m'\underset{1}{\|}n'} + {}'U_{m'}\,{}'U_{n'} = {}'u_{m'}\,{}'u_{n'}\frac{1}{\mathcal{R}_0^2}, \quad U^{s'}{}_{\underset{2}{\|}s'} + {}'U^{s'}\,{}'U_{s'} = \frac{1}{\mathcal{R}_0^2}\\
&B_{m'\underset{2}{\|}n'} + B_{m'}B_{n'} = h_{m'n'}\frac{1}{\mathcal{R}_0^2}, \quad B^{s'}{}_{\underset{2}{\|}s'} + B^{s'}B_{s'} = \frac{2}{\mathcal{R}_0^2}\\
&C_{m'\underset{3}{\|}n'} + C_{m'}C_{n'} = \left(m_{m'}m_{n'} + b_{m'}b_{n'} + {}'u_{m'}\,{}'u_{n'}\right)\frac{1}{\mathcal{R}_0^2}, \quad C^{s'}{}_{\underset{3}{\|}s} + C^{s'}C_{s'} = \frac{3}{\mathcal{R}_0^2}
\end{aligned}
\quad . \tag{A7}
$$

The definition of the quantities (A6) and equations (A7) can be found in [2]. By inserting these subequations of the Ricci into Einstein's field equations, the following are found for the Einstein tensor:

$$
G_{m'n'} = -g_{m'n'}\frac{3}{\mathcal{R}_0^2}, \quad \kappa p = \frac{3}{\mathcal{R}_0^2}, \quad \kappa\mu_0 = -\frac{3}{\mathcal{R}_0^2}. \tag{A8}
$$

7. The Friedman equations for the time-dependent models are as follows:

$$
\text{(I)} \quad {}'U_{m'} = \left\{0,0,0,-\frac{i}{\mathcal{R}}\right\}, \quad {}'U^{s'}{}_{\underset{1}{\|}s'} + {}'U^{s'}\,{}'U_{s'} = 0\,, \tag{A9}
$$

$$
\text{(II)} \quad {}'U_{m'} = \left\{0,0,0,\frac{1}{\mathcal{R}}\right\}, \quad {}'U^{s'}{}_{\underset{1}{\|}s'} + {}'U^{s'}\,{}'U_{s'} = 0\,. \tag{A10}
$$

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