ARG Austrian Reports on Gravitation

ARG-2005-01

KERR GEOMETRY V.
MORE ON FIVE DIMENSIONS

Rainer Burghardt*

Keywords: Kerr metric, static model, Lorentz transformation, anholonomic transformation

Contents

A static elliptical model is derived from the rotating Kerr geometry. Embedding this model in a five-dimensional flat space we are able to explain some features of the Kerr metric.

* e-mail: arg@aon.at, home page: http://arg.or.at/

1. INTRODUCTION

In Sec. 2 we will study a static elliptical model which can be deduced from the Kerr metric by an anholonomic transformation. This model is not Ricci flat. It has some similarity to the Schwarzschild model and to the Kerr model as well.

In Sec. 3 we will show that this model has a natural embedding in a flat five-dimensional space by utilizing the theory of double surfaces.

In Sec. 4 we investigate the invariance properties of the elliptical static model by introducing freely falling reference systems. We demonstrate that the force of gravity is compensated by the acceleration of these systems. Tidal forces can be experienced by the falling observers. These forces will be represented by second-rank tensors. They will satisfy covariant field equations.

2. THE STATIC MODEL

In a former paper [1] we have shown that the Kerr metric could be written as

$$ds^2 = dx^{12} + dx^{22} + \left[\alpha_R dx^3 + i\alpha_R \omega\sigma dx^4\right]^2 + a_S^2\left[-i\alpha_R\omega\sigma dx^3 + \alpha_R dx^4\right]^2 \qquad (2.1)$$

where

$$\begin{gathered} dx^1 = \alpha_S a_R dr, \quad dx^2 = \Lambda d\vartheta, \quad dx^3 = \sigma d\varphi, \quad dx^4 = idt \\ \alpha_R = \frac{A}{\Lambda}, \quad a_R = \frac{\Lambda}{A}, \quad \omega = \frac{a}{A^2} \\ \sigma = A\sin\vartheta, \quad a_S = \frac{\delta}{A}, \quad \alpha_S = \frac{A}{\delta}, \quad \delta^2 = r^2 + a^2 - 2Mr, \quad A^2 = r^2 + a^2, \quad \Lambda^2 = r^2 + a^2\cos^2\vartheta \end{gathered} \quad . \qquad (2.2)$$

A and r are the major and minor semi-axes of the confocal ellipses of the Boyer-Lindquist elliptical co-ordinate system. a is the eccentricity of the ellipses, ω the observer's angular velocity, and σ the observer's distance from the rotation axis.

In view of the fact that it is not possible to obtain from the Kerr metric a static metric by a Lorentz transformation the rotation is not attached to the geometry but geometrically implemented. The non-Lorentzian anholonomic transformation

$$\begin{aligned} &\Xi^3_{3'} = \alpha_R, \quad \Xi^3_{4'} = i\alpha_S\alpha_R\omega\sigma, \quad \Xi^4_{3'} = -ia_S\alpha_R\omega\sigma, \quad \Xi^4_{4'} = \alpha_R \\ &\Xi^{3'}_3 = \alpha_R, \quad \Xi^{3'}_4 = -i\alpha_S\alpha_R\omega\sigma, \quad \Xi^{4'}_3 = ia_S\alpha_R\omega\sigma, \quad \Xi^{4'}_4 = \alpha_R \end{aligned} \qquad (2.3)$$

differs from a Lorentz transformation by the gravitational factors α_S or a_S respectively. Transforming the 4-bein system with (2.3) and

$$\overset{m'}{e}_i = \Xi^{m'}_{m} \overset{m}{e}_i$$

the metric can be written as

$$ds^2 = \alpha_S^2 a_R^2 dr^2 + \Lambda^2 d\vartheta^2 + \sigma^2 d\varphi^2 + a_S^2 {dx^4}^2, \quad dx^4 = idt \quad . \tag{2.4}$$

and can be interpreted in a twofold way. i) The metric (2.4) is the anholonomic representation of the Kerr metric. The connexion coefficients have to be calculated by the inhomogeneous transformation law and this procedure leads to the field strengths of the Kerr metric again. ii) We leave it as written. Then (2.4) is the metric of a possibly new static elliptical model, which is not Ricci flat. It has some similarity to the Schwarzschild model and to the Kerr model as well. As the stress-energy-tensor does not seem to have a realistic interpretation it has no significance for the gravitation theory, but it could serve as a simplified model for studying some features of the Kerr geometry. The visible advantage of the second interpretation is that we are able to embed this metric in a five-dimensional flat space by using the theory of double surfaces [1,2,3]. The main concern of this paper is to infer the five-dimensional field equations from (2.4) and to perform the dimensional reduction.

3. THE EMBEDDING

To embed the metric (2.4) in a five-dimensional flat space with the rectilinear orthogonal co-ordinate system $X^{a'}$, a' = 0',1', … , 4' we start with a family of hyperspheres parametrized by

$$\begin{aligned} X^{3'} &= X \sin\varepsilon \sin\theta \sin\varphi \\ X^{2'} &= X \sin\varepsilon \sin\theta \cos\varphi \\ X^{1'} &= X \sin\varepsilon \cos\theta \qquad . \\ X^{0'} &= X \cos\varepsilon \cos i\psi \\ X^{4'} &= X \cos\varepsilon \sin i\psi \end{aligned} \tag{3.1}$$

The X are the radius vectors of the hyperspheres. The transformation to polar co-ordinates is performed with the matrix

$$D_a^{a'} = \begin{pmatrix} \cos i\psi & \cos\varepsilon\sin i\psi & -\sin\varepsilon\sin i\psi & 0 & 0 \\ -\sin i\psi & \cos\varepsilon\cos i\psi & -\sin\varepsilon\cos i\psi & 0 & 0 \\ 0 & \sin\varepsilon\cos\theta & \cos\varepsilon\cos\theta & -\sin\theta & 0 \\ 0 & \sin\varepsilon\sin\theta\cos\varphi & \cos\varepsilon\sin\theta\cos\varphi & \cos\theta\cos\varphi & -\sin\varphi \\ 0 & \sin\varepsilon\sin\theta\sin\varphi & \cos\varepsilon\sin\theta\sin\varphi & \cos\theta\sin\varphi & \cos\varphi \end{pmatrix}, \qquad (3.2)$$

where the sequence of indices is 4,0,1,2,3.

The flat space metric expressed by the above-defined polar co-ordinates reads as

$$ds^2 = dX^2 + X^2 d\varepsilon^2 + X^2\sin^2\varepsilon\, d\theta^2 + X^2\sin^2\varepsilon\sin^2\theta\, d\varphi^2 + X^2\cos^2\varepsilon\, di\psi^2 \ . \qquad (3.3)$$

Equ. (4.2) of paper [3] is extended by the last term of (3.3) and leads to an additional field strength on the hypersphere

$$X_{40}{}^4 = \frac{1}{X}, \quad X_{41}{}^4 = -\frac{1}{X}\tan\varepsilon . \qquad (3.4)$$

To derive a double-surface theory from this single-surface theory one has to introduce an additional projector

$$\mathfrak{p}_4^4 = \frac{X}{\rho_S} \qquad (3.5)$$

and we have to use the following expressions in the field equations

$$\left(\mathfrak{p}_4^4\right)^{-}\mathfrak{p}_{[4||1]}^{\;4} = \frac{1}{\rho_S}\rho_{S|1}, \quad \left(\mathfrak{p}_4^4\right)^{-1}\mathfrak{p}_{[4||2]}^{\;4} = \frac{1}{\rho_S}\rho_{S|2}, \quad \rho_S(r,\vartheta) = \Lambda\sqrt{\frac{2r}{M}\frac{r^2+a^2}{r^2-a^2}} \ . \qquad (3.6)$$

ρ_S is the curvature vector field of the radial integral lines of an elliptically squashed surface. This field connects a corresponding surface generated by the evolutes of the radial lines of the first surface.

By using (3.4) in $Y_{ab}{}^c = \mathfrak{p}_a^d X_{db}{}^c$ we get a new contribution to the connexion coefficients Y

$$E_{ab}{}^c = -\left[u_a E_b u^c - u_a u_b E^c\right], \quad E_b = \left\{-\frac{1}{\rho_S}, \frac{1}{\rho_S}\frac{v_S}{a_S}, 0,0,0\right\} \qquad (3.7)$$

E_1 is the force of gravity derived in [1]. The relation to the angles ε of the ascent of the surface mentioned above is $v_S = \sin\varepsilon,\ a_S = \cos\varepsilon$. By setting the parameter of rotation a to zero this quantity reduces to the force of gravity of the Schwarzschild theory. The Ricci tensor[1] derived from

$$\begin{gathered} \mathfrak{p}_a^g \mathfrak{p}_b^h R_{ghc}{}^d(X) = R_{abc}{}^d(Y) \\ R_{abc}{}^d(Y) = 2\left[Y_{[b\cdot c\cdot}{}^d{}_{|a]} + Y_{[b\cdot c\cdot}{}^f Y_{a]f}{}^d + Y_{[ba]}{}^f Y_{fc}{}^d + X_{fc}{}^d \mathfrak{p}^f_{[a|||b]} \right] = 0 \end{gathered} \tag{3.8}$$

includes the subequations

$$\left[E_{\underset{4}{b|||a}} - E_b E_a \right] + u_b u_a \left[E^c_{\underset{4}{|||c}} - E^c E_c \right] - 2\left[u_d E_b u^c - u_d u_b E^c \right] \left(\mathfrak{p}_f^d \right)^{-1} \mathfrak{p}^f_{[c|||a]} \ . \tag{3.9}$$

They decouple from the Ricci tensor by

$$E_{\underset{4}{b|||a}} - E_b E_a + E_b \frac{1}{\rho_S} \rho_{S|\underline{a}} = 0, \quad E^c_{\underset{4}{|||c}} - E^c E_c + E^c \frac{1}{\rho_S} \rho_{S|\underline{c}} = 0 \tag{3.10}$$

where $\underline{a} \neq 0$. A dimensional reduction leads to

$$R_{mn}(A) = -\kappa \left(T_{mn} - \frac{1}{2} g_{mn} T \right) \ . \tag{3.11}$$

The four-dimensional Ricci tensor and the four-dimensional connexion coefficients are defined by

$$\begin{gathered} R_{mn}(A) = A_{mn}{}^s{}_{|s} - A_{n|m} - A_{rm}{}^s A_{sn}{}^r + A_{mn}{}^s A_s \\ A_{mn}{}^s = B_{mn}{}^s + N_{mn}{}^s + C_{mn}{}^s + E_{mn}{}^s \end{gathered} \ , \tag{3.12}$$

$$\begin{gathered} B_{mn}{}^s = b_m B_n b^s - b_m b_n B^s, \quad B_n = \left\{ \frac{a_S}{\rho_E}, 0,0,0 \right\} \\ N_{mn}{}^s = m_m N_n m^s - m_m m_n N^s, \quad N_n = \left\{ 0, \frac{1}{\rho_H}, 0,0 \right\} \\ C_{mn}{}^s = c_m C_n c^s - c_m c_n C^s, \quad C_n = \left\{ \frac{a_S}{\rho_E} a_R^2, \frac{1}{\rho_E} a_R^2 \cot\vartheta, 0,0 \right\} \\ E_{mn}{}^s = -\left[u_m E_n u^n - u_m u_n E^n \right], \quad E_n = \left\{ \frac{1}{\rho_S} \frac{v_S}{a_S}, 0,0,0 \right\} \end{gathered} \ . \tag{3.13}$$

The field equations read as

[1] We make use of the formulae listed in paper [3]

$$
\begin{aligned}
&-\left[B_{n\underset{2}{||}m}-B_{n\underset{2}{||}s}b^{s}b_{m}+B_{n}B_{m}\right]-b_{n}b_{m}\left[B^{s}{}_{\underset{2}{||}s}+B^{s}B_{s}\right]\\
&-\left[N_{n\underset{2}{||}m}-N_{n\underset{2}{||}s}m^{s}m_{m}+N_{n}N_{m}\right]-m_{n}m_{m}\left[N^{s}{}_{\underset{2}{||}s}+N^{s}N_{s}\right]\\
&-\left[C_{n\underset{3}{||}m}+C_{n}C_{m}\right]-c_{n}c_{m}\left[C^{s}{}_{\underset{3}{||}s}+C^{s}C_{s}\right]\\
&+\left[E_{n\underset{4}{||}m}-E_{n}E_{m}\right]+u_{n}u_{m}\left[E^{s}{}_{\underset{4}{||}s}-E^{s}E_{s}\right]=-\kappa\left(T_{mn}-\frac{1}{2}g_{mn}T\right)
\end{aligned}
\tag{3.14}
$$

with

$$
\begin{aligned}
\kappa\left(T_{mn}-\frac{1}{2}g_{mn}T\right)=&-m_{m}m_{n}\left(M_{0}B_{0}+M_{0}C_{0}-M_{0}E_{0}\right)\\
&-b_{m}b_{n}\left(M_{0}B_{0}+B_{0}C_{0}-B_{0}E_{0}\right)\\
&-c_{m}c_{n}\left(M_{0}C_{0}+B_{0}C_{0}-C_{0}E_{0}\right)\\
&+u_{m}u_{n}\left(M_{0}E_{0}+B_{0}E_{0}+C_{0}E_{0}\right)\qquad .\\
&-\left(m_{m}m_{n}+b_{m}b_{n}\right)v_{S}^{2}\,\tilde{\Omega}^{s3}\tilde{\Omega}_{3s}\\
&+\left(m_{m}m_{n}+c_{m}c_{n}\right)v_{S}^{2}\,N_{2}C_{2}+2N_{(m}E_{n)}\\
&+\left(m_{m}m_{n}+u_{m}u_{n}\right)E_{1}\frac{1}{\rho_{S}}\rho_{S|1}
\end{aligned}
\tag{3.15}
$$

The first four lines on the right side of (3.15) include the generalized second fundamental forms of the physical surface. The two next lines are a contribution from the elliptical structure of the geometry and the last line results from the projection from the hypersphere of the single-surface theory to the elliptically squashed surface of the double-surface theory. With some algebra we simplify this expression to

$$
\kappa\left(T_{mn}-\frac{1}{2}g_{mn}T\right)=v_{S}^{2}\begin{pmatrix}
-2\tilde{\Omega}_{13}\tilde{\Omega}^{3}{}_{1} & & & \\
 & -2\tilde{\Omega}_{23}\tilde{\Omega}^{3}{}_{2} & & \\
 & & \tilde{\Omega}_{rs}\tilde{\Omega}^{rs}+2\tilde{F}_{s}\tilde{F}^{s} & \\
 & & & -\tilde{\Omega}_{rs}\tilde{\Omega}^{rs}-2\tilde{F}_{s}\tilde{F}^{s}
\end{pmatrix}-2F_{(m}E_{n)}\,. \tag{3.16}
$$

From this relation T_{mn} can be calculated. The quantities $\tilde{\Omega}$ and F are contributions from the BL-ellipsoid of revolution and they already occur in a flat elliptical system without rotation as explained in paper [5].

4. THE FREELY FALLING SYSTEM

Although we have simplified the rotating Kerr model to a static one, we hope to be able to extract further information from this model by introducing a freely falling system. We try to find an appropriate ansatz for the velocity of a freely falling observer in geometrical terms. Punsly [4] has made some efforts in studying this problem[2].

From the radial part of the Kerr line element [1]

$$dx^1 = \alpha_S a_R dr$$

we have separated the two quantities

$$\alpha_S = \frac{A}{\delta}, \quad a_R = \frac{\Lambda}{A} \tag{4.1}$$

where a_R is the elliptical factor discussed in [5] and

$$a_S = \alpha_S^{-1} = \frac{\delta}{\Lambda} = \cos\varepsilon \tag{4.2}$$

is related to the angle of the ascent of the radial curve of the physical surface [3]. We have already shown that a_R is related to the angular velocity ω of the rotating system by

$$a_R^2 = 1 - \omega^2\sigma^2$$

and that α_S can be interpreted as the Lorentz factor of a transformation to a freely falling system

$$\alpha_S = \frac{1}{\sqrt{1-v_S^2}}, \quad v_S = -\frac{r}{A}\sqrt{\frac{2M}{r}}\,. \tag{4.3}$$

v_S is the velocity of a freely falling observer incoming from infinity. v_S depends on the radial position but is independent of the angle ϑ. It differs from the Schwarzschild velocity by the ratio of the axis of the BL-ellipses. As $v_S = \sin\varepsilon$, the velocity v_S is defined by the ascent of the physical surface. The BL-ellipse at the waist of the physical surface reads as

$$\delta^2 = r^2 + a^2 - 2Mr = 0\,.$$

[2] The distance of the rotating observer from the axis of rotation we define by $\sigma = \sqrt{r^2 + a^2}\sin\vartheta$. Then σ is exactly the radius of curvature of the parallels of the BL-ellipsoid of revolution along which the observer travels. Due to this ansatz, we obtain results different from those of Punsly.

The solution

$$r_H = M + \sqrt{M^2 - a^2} \tag{4.4}$$

of this equation is called event horizon of the Kerr metric. If, as is the case in our simplified model, there is no rotation, (4.3) shows that an ingoing particle has approached the speed of light (v_H= -1) at the event horizon. The static horizon is prescribed by

$$r_0 = M + \sqrt{M^2 - a^2\cos^2\vartheta}\ , \tag{4.5}$$

which leads to

$$\delta_0 = a\sin\vartheta\ .$$

Rearranging this equation with the relations (2.2) we find for stationary observers that the orbital velocity [2] $\omega_{AC}\sigma$ has approached the speed of light $\omega_{AC}\sigma = 1$ at the static horizon. By another arrangement the relativistic sum of the orbital and radial motion has approached the speed of light

$$v_S^2 + \frac{1}{\alpha_S^2}\omega_{AC}^2\sigma^2 = 1\ . \tag{4.6}$$

A freely falling observer being dragged by the rotating frames is not able to exceed the static horizon. The above considerations encourage us to use the definitions (4.3) for a Lorentz transformation from a static system (2.4) to a freely falling system and likewise from a stationary system (2.1) to a freely falling one:

$$L_1^{1'} = \alpha_S,\ L_4^{1'} = i\alpha_S v_S,\ L_1^{4'} = -i\alpha_S v_S,\ L_4^{4'} = \alpha_S\ . \tag{4.7}$$

In previous papers [6,7] we have shown that the field equations and their subequations are invariant under Lorentz transformations. If we use a modified subsumption for the transformation law of the covariant derivation of tensors

$$\Phi_{m'\|n'} = L_{m'n'}^{m\ n}\Phi_{m\|n} = \left[\Phi_{m'|n'} - L_s^{s'}L_{m'|n'}^{s}\Phi_{s'}\right] - A_{n'm'}{}^{s'}\Phi_{s'}\,,\quad A_{n'm'}{}^{s'} = L_{n'm's}^{n\,m\,s'}A_{nm}{}^{s} \tag{4.8}$$

and if we define a new covariant derivative

$$\Phi_{m'\underset{1}{\|}n'} = \Phi_{m'|n'} - L_{n'm'}{}^{s'}\Phi_{s'}\,,\quad L_{n'm'}{}^{s'} = L_s^{s'}L_{m'|n'}^{s} \tag{4.9}$$

the Ricci tensor can be written as

$$R_{m'n'} = A_{m'n'}{}^{s'}{}_{\underset{1}{\|}s'} - A_{n'\underset{1}{\|}m'} - A_{r'm'}{}^{s'}A_{s'n'}{}^{r'} + A_{m'n'}{}^{s'}A_{s'}\,. \tag{4.10}$$

If the new subsumption is used, the subequations discussed in [3] and in the preceding Chapter turn out to be form invariant under passive Lorentz transformations. We obtain the same equations (3.14) wherein all the graded derivatives contain the Lorentz term L defined in (4.9). Now we consider the behavior of the subequations under active Lorentz transformations. We have to decompose the field equations with respect to the transformed tetrads

$$\begin{aligned} {}'m_{n'} &= L_1^{1'} m_{n'} + L_4^{1'} u_{n'} = \{1,0,0,0\}, \quad & {}'u_{n'} &= L_1^{4'} m_{n'} + L_4^{4'} u_{n'} = \{0,0,0,1\} \\ {}'b_{n'} &= b_n = \{0,1,0,0\}, & {}'c_{n'} &= c_n = \{0,0,1,0\} \end{aligned} \tag{4.11}$$

All quantities transformed by (4.7) have an additional time-like component. They are interpreted as tidal forces. We prefer for them the tensor representation. Examining the Lorentz term in (4.9) we get

$$\begin{gathered} L_{4'1'}{}^{4'} = G_{1'}, \quad L_{1'4'}{}^{1'} = Q_{4'} + G_{4'} \\ G_{m'} = \left\{ \alpha_S \frac{1}{\rho_S} \frac{v_S}{a_S}, 0, 0, -i\alpha_S v_s \frac{1}{\rho_S} \frac{v_S}{a_S} \right\}, \quad Q_{m'} = \left\{ 0, 0, 0, -\frac{i}{\rho_S} \right\} \end{gathered} \tag{4.12}$$

In the accelerated system the force of gravity (3.13) has the components

$$E_{4'1'}{}^{4'} = -E_{1'}, \quad E_{1'4'}{}^{1'} = -E_{4'}, \quad E_{m'} = \left\{ \alpha_S \frac{1}{\rho_S} \frac{v_S}{a_S}, 0, 0, -i\alpha_S v_s \frac{1}{\rho_S} \frac{v_S}{a_S} \right\}. \tag{4.13}$$

Although the acceleration G of the freely falling system is derived from the local rotation in the tangent space of the physical surface with $\alpha_S = \cos i\chi$, $i\alpha_S v_S = \sin i\chi$ and the force of gravity is derived from the ascent $\tan\varepsilon = v_S / a_S$ of the physical surface, both quantities are numerically equal. They enter the theory with opposite sign and compensate. Thus the freely falling observers can measure no radial acceleration. From now on we drop the primes of the indices.

From the two above relations we get

$$L_{mn}{}^{s} + E_{mn}{}^{s} = {}'m_m Q_n {}'m^s - {}'m_m {}'m_n Q^s . \tag{4.14}$$

If we go over to the tensor representation, we obtain the tidal forces

$$Q_{11} = Q_4, \quad Q_{22} = B_4, \quad Q_{33} = C_4 \ . \tag{4.15}$$

Splitting the quantity N in space-like (bars) and time-like parts (double-bars), we obtain

$$N_{mn}{}^{s} = \bar{N}_{mn}{}^{s} + \left['u_n N_m{}^{s} - 'u^s N_{mn} + 'u_m \left(N_n{}^{s} - N^s{}_n \right) \right] + \bar{\bar{N}}_{mn}{}^{s}$$

$$\bar{N}_{mn}{}^{s} = 'm_m \bar{N}_n 'm^s - 'm_m 'm_n \bar{N}^s, \quad \bar{\bar{N}}_{mn}{}^{s} = 'u_m \bar{\bar{N}}_n 'u^s - 'u_m 'u_n \bar{\bar{N}}^s . \tag{4.16}$$

$$\bar{N}_n = m_1 m_1 N_n, \quad N_{mn} = -'m_m m_4 m_1 N_n, \quad \bar{\bar{N}}_n = m_4 m_4 N_n$$

The graded derivatives for the accelerated systems are (α, β, γ =1,2,3)

$$\Phi_{\alpha \underset{2}{\wedge} \beta} = \Phi_{\alpha | \beta}, \quad \Phi_{\alpha \underset{3}{\wedge} \beta} = \Phi_{\alpha | \beta} - \bar{N}_{\beta\alpha}{}^{\gamma} \Phi_\gamma - B_{\beta\alpha}{}^{\gamma} \Phi_\gamma, \quad \Phi_{\alpha \wedge \beta} = \Phi_{\alpha | \beta} - \bar{N}_{\beta\alpha}{}^{\gamma} \Phi_\gamma - B_{\beta\alpha}{}^{\gamma} \Phi_\gamma - C_{\beta\alpha}{}^{\gamma} \Phi_\gamma \ . \tag{4.17}$$

Making use of the relation $m_m m_n + u_m u_n = 'm_m 'm_n + 'u_m 'u_n$ we get from (3.11) and (3.15) the field equations in terms of the accelerated observers

$$\begin{aligned} &\left[B_{\alpha \underset{2}{\wedge} \beta} - B_{\alpha \underset{2}{\wedge} \gamma} b^\gamma b_\beta + B_\alpha B_\beta \right] + b_\alpha b_\beta \left[B^\gamma{}_{\underset{2}{\wedge}\gamma} + B^\gamma B_\gamma \right] + \\ &+ \left[\bar{N}_{\alpha \underset{2}{\wedge} \beta} - \bar{N}_{\alpha \underset{2}{\wedge} \gamma} 'm^\gamma 'm_\beta + \bar{N}_\alpha \bar{N}_\beta \right] + 'm_\alpha 'm_\beta \left[\bar{N}^\gamma{}_{\underset{2}{\wedge}\gamma} + \bar{N}^\gamma \bar{N}_\gamma \right] + \left[\bar{\bar{N}}_{\alpha \wedge \beta} + \bar{\bar{N}}_\alpha \bar{\bar{N}}_\beta \right] + \\ &+ \left[C_{\alpha \underset{3}{\wedge} \beta} + C_\alpha C_\beta \right] + c_\alpha c_\beta \left[C^\gamma{}_{\underset{3}{\wedge}\gamma} + C^\gamma C_\gamma \right] + \\ &+ \left[N_{\alpha\beta \wedge s} 'u^s + N_{\alpha\beta} Q_\gamma{}^\gamma + N_\alpha{}^\gamma N_{\beta\gamma} - N^\gamma{}_\alpha N_{\gamma\beta} \right] + \left[Q_{\alpha\beta \wedge s} 'u^s + Q_{\alpha\beta} Q_\gamma{}^\gamma \right] = \kappa \left[T_{\alpha\beta} - \frac{1}{2} g_{\alpha\beta} T \right] \end{aligned} , \tag{4.18}$$

$$-\left[N_\alpha{}^\gamma{}_{\wedge\gamma} + N_\alpha{}^\gamma \bar{\bar{N}}_\gamma \right] - \left[Q_\alpha{}^\gamma{}_{\wedge\gamma} + Q_\gamma{}^\gamma{}_{\wedge\alpha} \right] = \kappa T_{\alpha n} 'u^n \quad , \tag{4.19}$$

$$\begin{aligned} &\left[\bar{N}_{\beta \underset{2}{\wedge} s} 'u^s + \bar{N}_\beta Q_{11} \right] + \left[B_{\beta \underset{2}{\wedge} s} 'u^s + B_\beta Q_{22} \right] + \left[C_{\beta \underset{3}{\wedge} s} 'u^s + C_\beta Q_{33} \right] - \\ &- \left[N_\beta{}^\gamma{}_{\underset{2}{\wedge}\gamma} - N^\gamma{}_{\beta \underset{2}{\wedge} \gamma} - N_\beta{}^\gamma N_\gamma \right] - 2 Q^\gamma{}_{[\gamma} \bar{\bar{N}}_{\beta]} = \kappa T_{m\beta} 'u^m \end{aligned} , \tag{4.20}$$

$$\left[\bar{\bar{N}}^\gamma{}_{\wedge\gamma} + \bar{\bar{N}}^\gamma \bar{\bar{N}}_\gamma \right] + \left[Q_\gamma{}^\gamma{}_{\wedge s} 'u^s + Q_{\alpha\beta} Q^{\alpha\beta} \right] = \kappa \left[T_{mn} 'u^m 'u^n - \frac{1}{2} T \right] . \tag{4.21}$$

Although we have reduced the rotating Kerr model to a static one, we can draw out from this model a lot of information that will be valid in the rotating version, too. In a former paper [4] we have studied more general transformations to accelerated systems, which could be also applied to the present model.

5. REFERENCES

1. Burghardt, R.; *Kerr geometry II. Preferred reference systems.* http:/arg.or.at/Wpdf/WKerr2.pdf
2. Burghardt, R.; *New embedding of the Schwarzschild geometry. I. Exterior solution.* http://arg.or.at/Wpdf/W5d.pdf and Sitzungsberichte der Leibniz-Sozietät **61**, 105, 2003
3. Burghardt, R.; *Kerr geometry III. A first step to five dimensions.* http:/arg.or.at/Wpdf/WKerr3.pdf
4. Punsly, B.; *A physical interpretation of the Kerr solution.* Journ. Math. Phys. **26**,2728, 1995
5. Burghardt, R.; *Kerr geometry I. A flat Kerr-like model.* http://arg.or.at/Wpdf/WKerr1.pdf
6. Burghardt, R.; *Kerr geometry IV. Accelerated systems.* http://arg.or.at/Wpdf/WKerr4.pdf
7. Burghardt, R.; *Freely falling observers.* http://arg.or.at/Wpdf/Wff.pdf